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ABSTRACT

The logical foundations of deduction and induction are outlined to form the rules for the construction of a set of tests of reasoning ability. Both deduction and induction involve the derivation of a conclusion from a set of premises. Deductive logic uses syllogisms and is abstract. Inductive logic is both empirical and abstract. Although inductive judgments cannot reliably tell us anything about particulars, we do in fact rely upon induction, and it constitutes our only possibility of making contact with the empirical manifold. Inductive reasoning may be used either to generate laws or to form judgments of probability. Item analysis data are presented, as are reliability coefficients. (Author/CTM)

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A LOGICAL APPROACH TO THE TESTING OF DEDUCTIVE AND INDUCTIVE ABILITIES

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ABSTRACT

This paper presents a logical analysis of deduction and induction with the purpose of demonstrating 1. that the two forms of argument are logically inseparable and hence psychometrically adjunctive, 2. that there are two complementary approaches to the testing of inductive abilities and that these approaches should always be coexistent, and 3. that a logical approach to the testing of deductive and inductive abilities is essential if the measuring instrument is to adjust with precision to the demands of the function for which it selects. On these three counts the research and development effort carried out at the Personnel Research and Development Center (PRDC) of the United States Civil Service Commission represents a refinement of traditional psychometric approaches to the measurement of reasoning abilities. This is especially the case in the context of induction, since the psychometric tradition has thus far failed to elucidate the two distinct complementary approaches to the testing of inductive abilities and consequently has neglected the development of tests designed to measure inductive abilities in the form of judgments of probability.

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INTRODUCTION

A belief appears to exist among some psychologists to the effect that deduction and induction as forms of argument are logically dichotomous. Actually it is the case in logic, as well as in philosophy of science, that deduction and induction express fundamentally the same logical process: the derivation of a conclusion from a set of premises. Peirce in fact refers to several forms of inductive judgments of probability as "probable deduction" (p. 191).

As Peirce himself would have been the first to emphasize, this is not to say that the two processes are logically univocal. However, their difference is one of truth-value rather than one of process. That is, in deductive logic the conclusion follows from a set of premises with logical necessity, whereas in inductive logic the conclusion does not follow necessarily from the premises but rather represents the statement of a certain degree of probability. Thus, inductive logic only serves the purpose of telling us "...how to calculate the value of this probability." (Carnap, 1974, p. 20)

The purpose of this paper is twofold. First, it intends to delineate the logical foundations of deduction and induction and to demonstrate the logical inseparability of both forms of argument. This inseparability will be empirically apparent in the results of experimental test administrations which are discussed in Chapter II. Second, it intends to demonstrate that a logical approach to the measurement of deductive and inductive abilities is job-based and alone makes possible the necessary convergence, in terms of reasoning process, between performance on the test and performance on the job.

A definition of this job-based logical approach may be briefly stated in the following terms: 1. precise, logical definitions of the reasoning processes required on the job must be established. These definitions thus far have not been elucidated with precision in the psychometric context primarily because little attention has been paid to the absolute relevance of logic to the testing of reasoning abilities. 2. In terms of reasoning process, the measuring instrument must adjust to the job with logical precision. Often this will entail modification of existing tests according to more precise deductive or inductive schemata.

In the context of test modification according to logical schemata, modifications carried out at the Personnel Research and Development Center (PRDC) of the U.S. Civil Service Commission will be discussed. This discussion will reveal two very important definitional points regarding the testing of deductive and inductive abilities: 1. psychometrically recognized marker tests for deduction should converge as exponents of formal deductive processes, and 2. there are two complementary approaches to the testing of inductive abilities which correspond to the two complementary stages of inductive argument: the generation of empirical (statistical) laws and the derivation of judgments of probability from an amalgamated premise which includes an empirical law. Since the inductive process is unitary, it is, to say the least, unlikely that a person would be called upon to reason inductively in one form while neglecting the other. Thus, it would appear that both forms should be measured conjointly.

Since the study of inductive logic presupposes the study of deductive logic, this paper will present the logical analysis of deduction first. Subsequently it will present an analysis of induction and an analysis of its logical and epistemic relation to deduction. Relevant psychometric considerations and discussions of experimental test administrations will be presented immediately after each of these analyses.

1. *Logical Analysis*

Many philosophers regard deductive formulae as being inherently tautological. In Kemeny's (1959) words:

> It [deduction] finds out certain facts which are contained in our statements, and adds nothing new (except in so far as this fact may be psychologically new to us; that is, we did not realize that we were in possession of this fact). (pp.112-113)

Deduction is indeed by its very nature a tautological process: the conclusions derived through its formulae are explicit elucidations of the actual content of the premises. As such--but assuming the truth of the premises--deduction represents an unassailable epistemic tool, for the conclusion follows apodictically from the premises.

An example of a deductive formula of the simplest type is:

$(x)\ (Px \supset Qx)$
Pa
Qa (Carnap, 1974, p. 17)

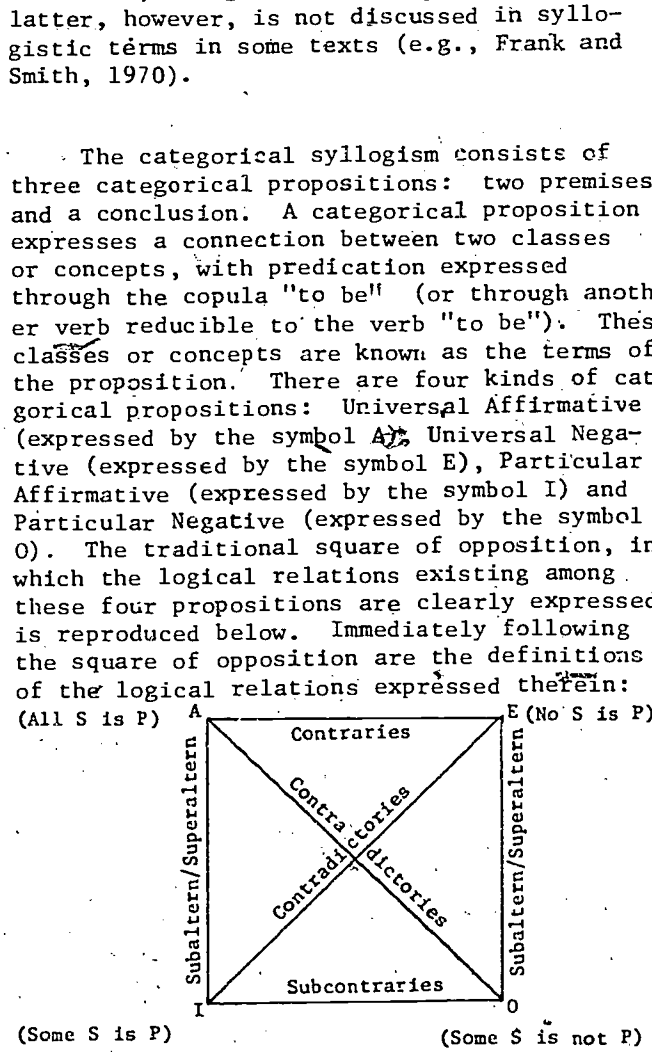
The first premise asserts that for all x, if x has the property P, then x also has the property Q. The term x at the beginning of the formula represents a universal quantifier and indicates that reference is being made to all cases of x (all deductive schemata include a universal premise). The symbol $\supset$ is a connective which indicates implication. The second premise asserts that a particular object a has the property P. From these premises the conclusion that object a has the property Q follows with logical necessity.

Broadly defined, therefore, deduction constitutes a demonstrative form of argument in which the conclusion has of necessity the same truth-value as the premises. Black (1970) thus rightly insists that the validity of the conclusion is axiomatic:

> There is no interval between considering and understanding the premises and wondering whether you have to accept the conclusion. *In* affirming the premises, you thereby affirm the conclusion. (pp. 20-21) [Similarly]: ...the counter-instances it envisages (p and *If p then q*, but possibly *not-q*) cannot be expressed without absurdity... (p. 22)

Basic deductive formulae are traditionally known as syllogisms. These may be of two kinds, categorical or compound. The latter, however, is not discussed in syllogistic terms in some texts (e.g., Frank and Smith, 1970).

The categorical syllogism consists of three categorical propositions: two premises and a conclusion. A categorical proposition expresses a connection between two classes or concepts, with predication expressed through the copula "to be" (or through another verb reducible to the verb "to be"). These classes or concepts are known as the terms of the proposition. There are four kinds of categorical propositions: Universal Affirmative (expressed by the symbol A), Universal Negative (expressed by the symbol E), Particular Affirmative (expressed by the symbol I) and Particular Negative (expressed by the symbol O). The traditional square of opposition, in which the logical relations existing among these four propositions are clearly expressed, is reproduced below. Immediately following the square of opposition are the definitions of the logical relations expressed therein:



Contradictories: Propositions cannot both be true and cannot both be false, i.e., from the truth of one we can validly infer the falsity of the other and from the falsity of one we can validly infer the truth of the other.

Contraries:[1] Propositions cannot both be true, i.e., from the truth of one we can infer the falsity of the other but the falsity of one leaves the truth-value of the other undetermined.

Superaltern/Subaltern:[1] The truth of the subaltern is included in the truth of the superaltern but the falsity of the superaltern leaves the truth-value of the subaltern undetermined. Conversely, the truth of the subaltern leaves the truth-value of the superaltern undetermined but its falsity determines the falsity of the superaltern.

Subcontraries:[1] Propositions cannot both be false at the same time, i.e., from the falsity of one we can validly infer the truth of the other but the truth of one leaves the truth-value of the other undetermined.

The categorical syllogism, as stated before, consists of three categorical propositions: two premises and a conclusion. These contain only three terms; of these, one is a comparative term which therefore appears in both premises and makes possible the derivation of the conclusion about the two terms being compared. The two terms being compared are called major and minor terms--the major term is the predicate of the conclusion and the minor term the subject of the conclusion.

The categorical syllogism has four figures, these being determined by the position of the middle term in the premises. Each figure has valid and invalid moods. The mood of a syllogism consists of the quantity (universal or particular) and quality (affirmative or negative) of the premises and the conclusion. The valid moods for each figure are listed below. Their validity is established by the axioms and theorems of the categorical syllogism which are given in Appendix A in order to provide ready accessibility to the logical foundations for each valid figure and mood.

It is, however, not necessary to analyze these axioms and theorems directly in order to follow through the deductive process of the syllogism. In Quine's (1972) words:

> As a practical method of appraising syllogisms, rules are less convenient than the method of diagrams. Indeed...we can apply the diagram test to a given argument out of hand, without pausing to consider where the argument may fit in the taxonomy of syllogisms. The diagram test is equally available for many arguments which do not fit any of the arbitrarily delimited set of forms known as syllogisms. (p. 91).

Valid Moods and Figures of the Categorical Syllogism

1. First Figure:

$$\begin{array}{l} M - P \\ \underline{S - M} \\ \therefore S - P \end{array}$$

valid moods: AAA, AAI, EAE, AII, EIO

2. Second Figure:

$$\begin{array}{l} P - M \\ \underline{S - M} \\ \therefore S - P \end{array}$$

valid moods: EAE, EAO, AEE, AEO, EIO, AOO

3. Third Figure:

$$\begin{array}{l} M - P \\ \underline{M - S} \\ \therefore S - P \end{array}$$

valid moods: AAI,[2] IAI, AII, EAO,[2] OAO, EIO

4. Fourth Figure:

$$\begin{array}{l} P - M \\ \underline{M - S} \\ \therefore S - P \end{array}$$

valid moods: AAI,[3] AEE, AEO, IAI, EAO,[2] EIO

NB: M = Middle Term	A = Universal Affirmative
S = Minor Term	E = Universal Negative
P = Major Term	I = Particular Affirmative
	O = Particular Negative

Encircled moods represent subaltern moods. These require implicit recognition of the added premise there are S.

[1] These relations clearly presuppose acceptance of the statement there are S. A discussion of this point is found in e.g., Quine (1972), pp. 84-85 .

[2] These moods require implicit recognition of the added premise there are M.

[3] This mood requires implicit recognition of the added premise there are P.

Compound or mixed syllogisms consist of either a conditional, alternative, or disjunctive proposition as the major premise, a categorical proposition as the minor premise, and a categorical proposition as the conclusion.

The major premise in the compound conditional syllogism consists of an antecedent and a consequent (*If p, then q*). From the affirmation of the antecedent in the minor premise follows the affirmation of the consequent in the conclusion, or, in the negative form, from the negation of the consequent in the minor premise follows the negation of the antecedent in the conclusion. The affirmative conditional and the negative conditional are often referred to in logic as Modus Ponens and Modus Tollens. Modus Ponens is essentially the same form of argument presented on p. 2. The syllogistic formulae[4] are as follows:

$$\begin{array}{l} p \supset q \\ p \\ \therefore q \end{array} \qquad\qquad \begin{array}{l} p \supset q \\ \sim q \\ \therefore \sim p \end{array}$$

where the symbol $\supset$ indicates a conditional statement and the symbol $\sim$ indicates negation. The same two syllogistic statements can be expressed theorematically[5] (e.g., Frank and Smith, 1970) as:

Theorem: $[(p \Rightarrow q) \wedge p] \Rightarrow q$

Proof: $[(p \Rightarrow q) \wedge p \wedge \sim q] \Leftrightarrow \sim(p \Rightarrow q)$

Theorem: $[(p \Rightarrow q) \wedge \sim q] \Rightarrow \sim p$

Proof: $[(p \Rightarrow q) \wedge \sim q \wedge p] \Leftrightarrow \sim(p \Rightarrow q)$

where the symbol => indicates a conditional statement, the symbol $\wedge$ indicates conjunction and the symbol <=> indicates equivalence.

The classical conditional sign is $\supset$ but in some current writing the conditional signs => and -> are used at least as frequently as the classical sign.

A more complex type of conditional syllogism is the pure conditional which involves a conditional statement in the minor premise and the conclusion as well as in the major premise. The logical conditional form remains the same.

$$\begin{array}{l} q \supset r \\ p \supset q \\ \therefore p \supset r \end{array} \qquad\qquad \begin{array}{l} r \supset \sim q \\ p \supset q \\ \therefore r \supset \sim p \end{array}$$

The major premise of the alternative syllogism consists of a statement of two alternants (*Either p or q*). The minor premise negates one of the alternants and the conclusion consequently affirms the other alternant. The syllogistic formula is expressed as:

$$\begin{array}{l} p \vee q \\ \sim p \\ \therefore q \end{array}$$

where the symbol V indicates alternation.

In theorematic terms:

Theorem: $[(p \vee q) \wedge \sim p] \Rightarrow q$

Proof: $[(p \vee q) \wedge \sim p \wedge \sim q] \Leftrightarrow \sim(p \vee q)$

A disjunctive syllogism is the same in form as an alternative syllogism except that the major premise explicitly states the mutual exclusion of the alternants, the minor affirms one of the alternants, and the conclusion negates the other alternant. The syllogistic formula is therefore expressed as:

$$\begin{array}{l} \sim(p \wedge q) \\ p \\ \therefore \sim q \end{array}$$

The deductive formulae presented in the foregoing discussion, as well as other more complex forms of deductive reasoning, are, to

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[4] Examples of all compound syllogisms in propositional form are given in Appendix B.

[5] Black's viewpoint (presented on p. 2 of this paper) on the axiomatic quality of these arguments should be recalled here. Essentially these arguments are axiomatic rather than theorematic and the proof presented for each theorem is really an explicit statement of the logical absurdity involved in affirming the truth of the premises and denying the necessity of the conclusion.

reiterate the definitional points made in the opening remarks of this section, axiomatic in quality. Deductive conclusions are apodictic.

However, as Carnap (1974) puts it,

> It is true that the laws of logic and pure mathematics (not physical geometry which is something else) are universal, but they tell us nothing whatever about the world. They merely state relations that hold between certain concepts, not because the world has such and such a structure, but only because those concepts are defined in certain ways. (p. 9). [Further] The laws of logic and pure mathematics, by their very nature, cannot be used as a basis for scientific explanation because they tell us nothing that distinguishes the actual world from some other possible world. When we ask for the explanation of a fact, a particular observation in the actual world, we must make use of *empirical* laws. They do not possess the certainty of logical and mathematical laws, but they do tell us something about the structure of the world. (pp. 11-12).

Thus, as the relevance of deduction disappears when in confrontation with the existent as such, the relevance and inescapability of induction becomes prominent. We might apply Unamuno's ingenious words on Hegel to the failure of deduction vis à vis the existent: "Hegel, a great framer of definitions, attempted with definitions to reconstruct the universe, like that artillery sergeant who said that cannon were made by taking a hole and enclosing it with steel." (p. 5).

Nevertheless, induction is in a way the deductive process as transformed by the epistemic needs which arise due to the existence of the empirical. Furthermore, the relevance of deduction as reasoning process is as absolute as that of induction: it is perhaps primordial to question the validity of universal premises, but it is equally necessary to discover the implications of given universals or of artificially constructed universals. Deduction and induction are, in one word, inseparable.

Before we begin to elucidate these comments and to analyze induction in itself, we will discuss the testing of deductive abilities and present some psychometric considerations which stem directly from the logical analysis of deduction presented above.

2. *The Testing of Deductive Abilities*

The primary purpose of the testing of reasoning abilities for employment selection is to connect two situations, the testing situation and the job situation, to the point of essential convergence. In other words, in terms of reasoning process, the testing situation must reflect the job situation with precision. Otherwise prediction is inaccurate and the test fails, at least partially, in its purpose.

It follows that the issue of whether ability is acquired or innate[6] does not substantially affect the concept of testing. In fact, whether or not the ability exists as such, predictive access to the specific performance of a person on the job is relevant. This predictive access is possible only when, as stated above, the measuring instrument adjusts with precision to the demands of the job.

These comments may appear redundant in a discussion of testing, but they certainly are not: there are degrees of connectedness, in terms of reasoning process, between performance on the job and performance on the test. What we are advocating here is that unless the degree of connectedness or convergence is *essential*, the test will bifurcate from the job.

This is the very basis of a logical approach to the testing of deductive (and inductive) abilities. The deductive process is unitary and conforms to the laws of logic. Hence, if deductive reasoning forms part of the job definition, the measuring instrument for deductive abilities must conform absolutely to deductive schemata, i.e., on the job as well as on the test correct deductions are crucial.

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[6] The term innate in any case may be interpreted in the Platonic sense (e.g. in the *Meno*) in which the innateness of the trait does not negate the relevance of the learning process. In fact, the learning process is the *sine qua non* for the actualization of the trait.

One test-part in the deductive test of the Professional and Administrative Career Examination (PACE) of the Federal testing program is modeled after Guilford's Inference Test, an adaptation of which is listed in the factor-analytic research reported by French, Ekstrom and Price (1963) and by Ekstrom (1973).

In the French, Ekstrom and Price manual and in the Ekstrom synthesis Factor Rs is called Syllogistic Reasoning due to the possibility of a more generic or factorially plural applicability of the term deduction.[7] The term is certainly appropriate although the term deduction in any case necessarily entails the deductive formulae known traditionally as syllogisms.

One of the findings of the job-oriented logical research project carried out in PRDC was that it would be necessary to modify the Guilford Inference Test according to deductive schemata.

It was found first of all that the Inference Test as it appears in the French, Ekstrom and Price manual does not strictly conform to deductive processes. In fact, although the Inference Test converges statistically with tests of syllogistic reasoning, speaking strictly from the logical standpoint the Inference Test is not a test of syllogistic reasoning, since it does not exhibit deductive schemata. Furthermore, the test often appears to present a correct answer in terms of a repetition of the information contained in one proposition, which contradicts the very definition of the deductive process.

Thus, although the format of the Inference Test was preserved in the PACE, construction criteria were developed according to deductive schemata. The format of the Inference Test seemed preferable to that of simple syllogistic formulae once again in terms of the job, i.e., in jobs covered by the Federal testing program, deductive processes are carried out primarily from data presented in ordinary language.

The deductive schemata utilized in the construction criteria include all the deductive formulae presented in section 1 of this chapter, except the subaltern moods of the first, second and fourth figures of the categorical syllogism: since the universal conclusion is warranted by the premises in these moods, a particular conclusion, although legitimate, constitutes an omission of part of the quantitative evidence.

The construction criteria also included a precise logical plan for the construction of distracters within the specific context of each deductive formula. In the case of categorical syllogisms, these construction criteria included all possible invalid quantitative and qualitative manipulations of premises and conclusions, e.g., invalid alterations of quantity and quality in the two premises and the conclusion according to the square of opposition, and valid and invalid conversions, obversions, contrapositions, obverted conversions and inversions of the premises.[8] In the case of compound syllogisms, invalid logical conclusions were utilized for each form (e.g., $[(p \Rightarrow q) \wedge p] \Rightarrow \sim q$ for Modus Ponens) and, once these were exhausted, patterns were established for invalid negations of propositional terms and for the conversion of premises.

These construction criteria were studied through the administration of three experimental test series to competitors for Federal professional and administrative positions. The three test series represent a total of 60 categorical syllogisms and 30 compound syllogisms.

As reflected in high point-biserial correlations, data from these administrations uniformly revealed an impressive discrimination between the upper- and lower-scoring groups. In other words, the upper-scoring group, taken as a whole, demonstrated a consistent ability to follow through the deductive process, whereas the lower-scoring group, taken as a whole, demonstrated a

[7] The manual expresses it in the following terms: "Since the name Deduction may better describe another factor or factors and since the tests most consistently loading this factor are rather specific, it seemed most unambiguous to name it Syllogistic Reasoning." (p. 37). In consonance with this reasoning and further along its lines, the Ekstrom synthesis defines Syllogistic Reasoning as "...probably a sub-factor of the ability called Deduction by Thurstone." (p. 55).

[8] Propositional equivalence (conversion, obversion, obverted conversion, contraposition and inversion) is defined in Appendix C.

consistent failure to follow through the fundamental deductive process in leaning heavily towards distracters representing propositional equivalents of the premises or invalid deductions from the premises. Frequency distributions and scatterplots of the p-values and point-biserial r-values for the three series are given in Appendices D through F.

In terms of test reliability the experimental data presented in Table 1 below also support the postulate that a deductive test constructed strictly according to deductive formulae constitutes a refined approach to the measurement of deductive abilities. The KR-20 values for three 30-question experimental series constructed according to deductive formulae are compared to the KR-20 value of a 30-question series of unmodified Inference.

Table 1

	KR-20
Series 19013 (unmodified Inference)	.79
Series 21013 (modified Deduction)	.87
Series 21014 (modified Deduction)	.88
Series 19012 (modified Deduction)	.87

3. *Summary*

Deduction as logical process is a demonstrative form of argument in which the truth-value of the premises is the same as the truth-value of the conclusion. That is to say, it is impossible to accept the premises without accepting the conclusion.

However, as an epistemic approach to the empirical manifold deduction is insufficient and the complementary need for nondemonstrative conclusions--i.e., determinations as to degrees of probability--becomes apparent. Nondemonstrative argument, induction, is discussed in Chapter II.

As follows from the discussion presented in section 2, a test of deductive abilities conforms to its identity only when it is constructed strictly according to deductive schemata.

Since the deductive process is unitary, conformance to deductive schemata in the construction of the test alone makes possible the necessary convergence, in terms of reasoning process, between performance on the job and performance on the test, i.e., on the job as well as on the test deductions are correct only when they reflect these schemata.

Lastly, the modification of the Inference Test according to deductive formulae ipso facto makes the marker tests listed in the 1973 Ekstrom synthesis (tests of Syllogistic Reasoning and the Inference Test) converge as exponents of formal deductive processes.

INDUCTION

1. *Logical Analysis*

In contradistinction to deduction and as a form of argument, induction is nondemonstrative. In other words, as we have seen, it is impossible for a deductive conclusion, Qa, to be false if the premises, $(x)\ (Px \supset Qx) \wedge Pa$, are true. However, even if the premises are true, it is possible for the conclusion Qa to be false if it is inductively established.

In Black's (1970) words:

> [The name induction may be used to] cover all cases of nondemonstrative argument, in which the truth of the premises, while not entailing the truth of the conclusion, purports to be a good reason for belief in it. (p. 57). [and] Induction stands here for any kind of nondemonstrative argument whose conclusion is not intended to follow from the premises by sheer logical necessity. The negation of an inductive conclusion is compatible with the amalgamated premise (the conjunction of all the reasons offered in support of the conclusion). (p. 137).

There are two classes or levels of inductive reasoning. One, a priori probability in itself, which deals with "the incidence of certain features in the total of all possible mathematical combinations of a given sort" (Ayer, 1972, p. 30), will not be explored as such in this paper. As stated at the end of Chapter I and because of its ultimately psychometric orientation, this paper is essentially concerned with inductive reasoning as it functions within the empirical context. A priori probability if and when applied to the empirical necessitates certain assumptions and added empirical meanings which go beyond its definition. As Ayer (1972) has put it:

> Mathematically, the alternatives 1-6, in the case of the dice, are equally probable in the entirely trivial sense that in the series 1-6 each of them occurs just once. But clearly this triviality cannot be applied to any actual game, unless one makes suitable assumptions about actual frequencies, and then introduces a new notion of probability, or at least gives the old notion a new application which will need to be defined. (p. 30).

The second class or level is constituted by what we might call statistical/logical[9] judgments of probability. The basic schema of these judgments may be stated simply as:

$rf\ (Q,P) = .8$
Pa
Qa (with a probability of .8)

(Carnap, 1974, p. 37)

The first premise states that the relative frequency of Q with respect to P is .8. The second premise states that a certain particular a has the property P, and the third statement asserts that this particular a has the property Q with a probability of .8. Essentially the point is that the conclusion about a regarding Q, as all inductive conclusions, must be derived in advance of possession of *all* relevant knowledge about the conjunctive incidence of Q and P. As stated in the concluding remarks of Chapter I, Section 1, the inescapability of deriving these conclusions arises from the epistemic confrontation with the empirical manifold which cannot be reduced to the universal laws which operate in deductive judgments.

As regards the conclusion Qa, the probability of .8 is affirmed of the statement itself. In other words, what is affirmed is that statement Qa, with regard to the evidence presented in the premises, has a probability of .8. This point is extremely important: the derivation of statement Qa on the basis of the premises represents a logical statement of probability, i.e., what is expressed in affirming it is a logical relation between the evidence and the conclusion. As such this affirmation represents an analytic statement or a statement of logical probability (i.e., degree of confirmation), not an empirical statement. In Carnap's (1974) words, "It is analytic because no empirical investigation is demanded. It expresses a logical relation between a sentence that states the evidence and a sentence that states the hypothesis." (p. 35).

By contrast, the premise $rf(Q,P)=.8$ represents a statistical law, an empirical statement. The formula taken as a whole, therefore, exhibits both concepts of probability, statistical or empirical and logical or analytic: the first premise is empirical, but the derivation of the conclusion, which is to say the inductive judgment itself, is analytic or logical.

The derivation of the statistical law is, needless to say, a matter of crucial importance, although as such it is independent from the issue of the inductive judgment expressed by the formula. Certain obvious, basic questions regarding the derivation of the law would be: Was it derived on the basis of the frequency of Q,P in an observed sample? Or on the basis of the frequency of Q,P in the total population? If the former is the case, then, as Carnap (1974) states,

> Only the value of the frequency in the sample is known. The value of the frequency in the population is not known. The best we can do is make an *estimate* of the frequency in the population. This estimate must not be confused with the value of the frequency in the sample. In general such estimates should deviate in a certain direction from the observed relative frequency in a sample. (p. 38).

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9 It is important to take note of the use of the word "logical" here. Although we have distinguished this type of inductive judgment from the a priori calculus of chances itself, this does not mean that there are no elements of a priorism in what we call statistical/logical inductions. This point is discussed immediately below.

Carnap discusses the question and presents a number of techniques for making these estimates in *The Continuum of Inductive Methods* (1952).

Beyond these basic and essentially answerable questions there are fundamental issues regarding the general cadre of induction which have literally plagued the philosophy of induction and which are actually regarded as unresolvable. It is certainly essential for anyone concerned with induction, at any level, to be fairly well acquainted with these issues. A relatively brief discussion will be presented here. More extensive presentations may be found in, e.g., Ayer, 1972; Black, 1970; or Barker, 1967.

Hume's famous analysis presented in *A Treatise on Human Nature* and in *An Enquiry Concerning Human Understanding* is at the root of the inductive impasse. Hume maintained that through empirical observation no epistemic effort can succeed in discovering more than contiguity and an internal habit of association. One of the most concise expositions of Hume's impact is presented by Ayer (1972) in his book *Probability and Evidence*. As he puts it,

> There is no such thing as a synthetic necessary connection between events. These are not, of course, the terms in which Hume puts it, but this is what it comes to. No matter what events *A* and *B* are, if *A* is presented to us in some spatio-temporal relation to *B*, there is nothing in this situation from which we could validly infer, without the help of other premisses, that events of the same type as *A* and *B* are connected in the same way on any other occasion. There is no such thing as seeing that *A* must be attended by *B*...
> [And] ...clearly the inference from the premiss 'Events of the type *A* and *B* have invariably been found in conjunction,' or to put it more shortly, 'All hitherto observed *As* bear the relation *R* to *Bs*,' to the conclusion 'All *As* bear the relation *R* to *Bs*,' or even to the conclusion 'This *A* will have the relation *R* to some *B*,' is not formally valid. There is what we may call an inductive jump. (p.4).

One might recall here Bertrand Russell's celebrated witticism on induction: "The man who has fed the chicken every day throughout its life at last wrings its neck instead, showing that more refined views as to the uniformity of nature would have been useful to the chicken." (p. 115).

It must be observed that the inductive impasse affects deduction as well as induction. The empirical possibility of a universal law, which, as we have seen in Chapter I, constitutes, of necessity, one of the premises in deductive judgments, is negated by this argument. The universal law can only exist as a law of logic or of pure mathematics, or else it must be inductively established (in order to exist as an empirical law). Thus, deductive conclusions are logically axiomatic but would be epistemically unassailable if and only if the truth of the universal evidence were firm and unassailable.

Analogously, as we have seen, the derivation of an inductive conclusion is in itself analytic and is only dismantled epistemically if it seeks to subsume the empirical, that is to say, if it intends to refer to particular events and to be empirically predictive: From the logical standpoint, event Qa cannot be said to have probability in itself. What the inductive judgment affirms logically is that the probability value of proposition Qa is affected and defined as .8 by the amalgamated premise $rf\ (Q,P) = .8 \wedge Pa$. It does not thereby affirm that particular event a in itself possesses a .8 probability of having the property Q.

The difference between deduction and induction is hence that if universal propositions were empirically possible, deductive judgments would constitute reliable epistemic tools; inductive judgments, by contrast, can never constitute a reliable epistemic tool. In other words, they do not reliably tell us anything about particulars.

Ironically, however, we do in fact rely upon induction. Indeed, in its very limitations and in spite of its inherent unreliability, induction constitutes our only possibility of making contact with the empirical manifold. Epistemically, the amalgamated inductive premise, perhaps against our superior logical sense, constitutes grounds for tentative belief. As Black (1970) puts it, "Standard induction is preferable to soothsaying because we know that it will work (will approach limiting values in the long run) if *anything* will." (p. 79). Or, in Barker's (1967) words,

> ...given the evidence that we have, we must draw whatever nondemonstrative conclusions we can from it. To contend that given evidence cannot be employed in nondemonstrative argument unless there is further positive evidence that the given evidence is not misleading is to embark on a vicious infinite regress, a regress which would destroy the possibility of there being any nondemonstrative arguments at all. (pp. 23-24).

In a sense, the inductive problem is a pseudoproblem, for it is impossible even to think about perceivable phenomena without being caught into probabilities and uncertainties. Once again it is pertinent to recall Unamuno's criticism of the Hegelian artillery sergeant and admit that the empirical manifold, far from being susceptible to definite enclosure, exceeds and overflows all formulae. Whereas inductive evidences are in themselves summative, they are, as Ayer (1972) declares, "...marginally predictive... [and] for the most part it is their predictive element which gives them their point." (p. 50).

In recognizing the pseudoproblematic character of the inductive impasse, Black (1970) states:

> If induction is by definition nondeductive and if the demand for justification is, at bottom, that induction be shown to satisfy conditions of correctness appropriate only to deduction, then the task is certainly hopeless. But to conclude, for this reason, that induction is basically invalid or that a belief based upon inductive grounds can never be reasonable is to transfer, in a manner all too enticing, criteria of evaluation from one domain to another domain, in which they are inappropriate. Sound inductive conclusions do not follow (in the deductive sense of "follow") from even the best and strongest set of premises (in the inductive sense of "strongest"); there is no good reason why they should. (p. 83).

In what is perhaps an attitude of compromise one is forced to affirm that the ultimate purpose of induction is to establish the tentative reliability of hypotheses upon which we are forcibly called to act. We might conclude with Russell that "...our instincts certainly cause us to believe that the sun will rise tomorrow, but we may be in no better a position than the chicken which unexpectedly has its neck wrung." (p. 115).

2. *The Testing of Inductive Abilities*

As stated in our discussion on the testing of deductive abilities, the primary function of the testing of reasoning abilities for employment selection is to connect two situations, the testing situation and the job situation, to the point of essential convergence. That is to say, the testing situation must reflect the job situation with precision. Otherwise prediction is inaccurate and the test fails, at least partially, in its purpose.

The inductive process is, as we have seen, unitary. It conforms to laws in the derivation of conclusions of probability (which we, as knowing subjects, interpret as predictive). It must be made explicit here that we do not mean to exclude from this unitary definition the summative process which generates the statistical law. Obviously one must call these investigations inductive. Indeed classical writers did so. Mill, for example, defined induction as "generalization from experience." (p. 64). This point is of essential relevance to psychometrics and will be discussed below.

Thus, given that inductive reasoning forms part of the job definition and since inductive arguments conform to logical laws, the measuring instrument for inductive abilities must conform to inductive schemata, i.e., on the job as well as on the test correct inductions are crucial.

This job-oriented logical approach to the testing of inductive abilities constituted a major research effort in PRDC. The aim of this research project was to determine which existing psychometric approach to the testing of inductive abilities converged to the most precise degree with inductive job requirements and whether or not the psychometric approach in question was susceptible to refinement according to the schemata of inductive logic.

In the psychometric tradition there are two approaches to the testing of inductive abilities: One approach has been traditionally attached to the concept, or inductive stage, of generating laws from observed particulars. The other approach has been traditionally attached to the concept of inductive judgment (determinations regarding probability) discussed in section 1 of this chapter.

By far the most prominent approach is the former. It is found for example in the French, Ekstrom and Price (1963) Letter Sets and Figure Classifications tests. The second approach, i.e., the testing of the inductive abilities entailed in carrying out judgments of probability, is literally not prominent in the psychometric literature. It is found for example in the Watson-Glaser Inference Test although this test does not exhibit (and does not intend to exhibit) a purely inductive form. That is to say, the test-taker must discern, in some cases, the probability established by a certain body of evidence and choose an alternative which expresses this probability from among a set of conclusions which offer alternative statements of falsehood and truth. As Ross (Note 1)correctly indicates, "The items which require T [True] and F [False] responses are apparently deductive items, while the items requiring PT [Probably True] and PF [Probably False] responses are probably inductive." (p. 12)

Accordingly, in PRDC, research in the realm of induction became immediately concerned with the development of a test part to attempt to measure inductive abilities in the form of pure judgments of probability. The linguistic medium was utilized because the professional and administrative jobs for which the test selects require the utilization of this medium in carrying out inductive judgments.

A nonlinguistic test was preserved, in adherence to traditional psychometrics, to test inductive abilities at the level of law-generative judgments. It is relevant to note, however, that although the nonlinguistic medium has traditionally been utilized to test law-generative inductive abilities, this medium does not constitute a corollary of the testing of these abilities. Conceivably these abilities could be tested through the linguistic medium. The schema would be one of the form:

P_1
P_2
.
.
.
.
Pn
Therefore (probably)
K

(Black, 1970, p. 147)

As Black (1970) points out, "Here, the qualifier "probably" may be conceived to be attached, as shown, to the "Therefore" (the sign of illation)..." (p. 147).

The choice of an inductive test depends directly on which form of argument (law-generative inductive reasoning or inductive reasoning in the form of judgments of probability) the job will primarily necessitate. However, it is likely that in most domains both forms will be called for equally. One could safely venture to say that although we, as reasoning subjects, sometimes carry out one form of argument without attention to its complementary form, it is extremely unlikely that this would be the case universally. Thus most testing situations, if at all intended for specific purposes, would attend to both needs.

The two definitions involved in these two psychometric approaches have not thus far been elucidated with precision in the psychometric literature mostly because little attention has been given to the relevance of logic to the testing of reasoning abilities. In turn the lack of contact with logic may have unwittingly prompted, in many cases, the choice of only one test when it is more than likely that the use of both tests would have been more appropriate. The fundamental point is that the two types of test exhibit different psychometric definitions: one is conceived in terms of the ability to generate the statistical laws which will later be used in inductive arguments; the other is conceived in terms of the ability to carry out the inductive argument itself without attention to the generation of the premise which serves as statistical law.

The test designed in PRDC to measure inductive abilities in the form of judgments of probability was designed according to three principal schemata. These are:

I. $rf\ (Q,P) = .x$
Pa
Qa (with a probability of $.x$)

(Carnap, 1974, p. 37)

II. Most P's each bear R to some Q or other.
Some S's are P's (and no Q's to which they bear R are observed).
Therefore (probably) there are some Q's (not observed) to which these S's bear R.

(Barker, 1967, p. 97)

III. Of all the things that are M, $\frac{m}{n}$ are P.
a is an M.
Therefore (with a probability of $\frac{m}{n}$) a is a P.

(Barker, 1967, p. 70)

These schemata express the basic logic of inductive argument. Nevertheless the test deviates from traditional psychometrics and represents a refinement thereof in that it is constructed in the form of *pure* judgments of probability. Thus, in contrast with the Watson-Glaser Inference Test, alternative answers were constructed according to a logical plan whereby in every case the test taker is asked to discern, on the basis of the evidence, a correct statement of probability from incorrect statements of probability. In terms of the job considerations discussed in this paper, this construction criterion acquires essential significance: being able to discern a valid conclusion of probability from invalid conclusions of probability is crucial on the job.

The questions are also innovative in that they were constructed with attention to the logic of induction itself. Thus, the statement "With respect to this evidence" was included in lead form before the five alternative statements of probability. If this qualifying statement is not included, statements of probability exceed the limits of logic and confuse prediction with truth-value.

These construction criteria according to purely inductive schemata do not entail logical isolation from deductive reasoning. In fact this is far from being the case for, as we have seen, inductive and deductive judgments differ in terms of truth-value rather than in terms of reasoning process. This point will be discussed again below.

Three test series of questions consisting of judgments of probability constructed according to the schemata and criteria discussed above were administered experimentally to competitors for Federal professional and administrative positions. One of the test series consisted of 30 questions; the other two of 20 questions each.

As reflected in high point-biserial correlations, data from these administrations revealed, with very few exceptions, a desirable discrimination between the upper- and lower-scoring groups. In other words, the upper-scoring group, taken as a whole, demonstrated a consistent ability to follow through the inductive process, whereas the lower-scoring group, taken as a whole, demonstrated a consistent failure to follow through the inductive process in leaning heavily towards distracters representing invalid judgments of probability. Frequency distributions and scatterplots of the p-values and point-biserial r-values for the three series (Series 21015, 26006 and 26007) are given in Appendices G through I.

In terms of test reliability, the data presented in Table 2 below also support the postulate that the linguistic test of inductive abilities, as modified according to the logical schemata and construction criteria discussed above, constitutes a reliable approach to the measurement of inductive abilities. The KR-20 values for three experimental series of these questions are compared to the KR-20 value for a series of 30 non-linguistic questions of the law-generative form.

Table 2

	KR-20 Obtained	KR-20 Estimated for 30-item test[10]
Series of 30 nonlinguistic questions of the law-generative form	.838	
Series of 30 experimental linguistic questions of judgments of probability (Series 21015)	.796	
Series of 20 experimental linguistic questions of judgments of probability (Series 26006)	.740	.810
Series of 20 experimental linguistic questions of judgments of probability (Series 26007)	.676	.758

In terms of subtest intercorrelations, the experimental test corroborated its logical groundwork. Induction, in the form of judgments of probability, is inseparable from deduction, as this paper has often reiterated. In fact, the generalized schemata are almost identical. If we recall Modus Ponens and compare it to the generalized schema presented for induction, we realize that the formulae differ only insofar as the first premise may represent universality or relative frequency:

$(x)\ (Px \supset Qx)$	$rf(Q,P)=.8$
Pa	Pa
Qa	Qa (with a probability of $.8$)

Thus, induction is in a way, as indicated in Chapter I, the deductive process as transformed by the epistemic needs which arise upon confrontation with the empirical manifold.

In a testing situation, therefore, it is desirable to obtain correlated results between inductive and deductive subtests (especially if the inductive subtest consists of judgments of probability drawn from linguistic data).

Accordingly, as shown in Table 3, the correlation obtained for the 30-question experimental series of linguistic inductive judgments of probability and a deductive subtest is higher (.64) than the correlation (.52) obtained for the experimental series and an inductive subtest consisting of 30 questions of the nonlinguistic law-generative form. It is also relevant to note that this correlation of .52 is lower than the correlation (.63) obtained for the experimental series with a verbal subtest. This is explainable (and desirable) not only from the standpoint of the testing medium, which was in both cases the linguistic medium, but also from the logical standpoint: the 30-question verbal subtest contains 15 questions which are based on the logical concept of propositional equivalence discussed in Chapter I; propositional equivalence, needless to say, entails a deductive inferential process. Lastly, the correlation (.61) obtained for the deductive subtest and the nonlinguistic law-generative inductive subtest should be interpreted in light of the fact that the 30-question deductive subtest includes 15 questions of numerical deduction.

Table 3

Subtest intercorrelations for operational deductive, verbal and nonlinguistic law-generative inductive subtests with experimental series of linguistic inductive judgments of probability (Series 21015)

N= 1,473

	Series 21015	Operational Subtests I	II	III
Series 21015		.64	.52	.63
I Deduction			.61	.63
II Nonlinguistic law-generative Induction				.49
III Verbal				

[10] These estimates were made using the Spearman-Brown formula for estimating the reliability of a lengthened test (Nunnally, 1967, p. 223).

Once again, therefore, this research effort finds itself in convergence with Ross' insights. Ross (Note 1) concludes on this issue that "...it is possible that the inductive-deductive dichotomy is a poor way to conceptualize reasoning abilities. Perhaps the two are inseparable." (p. 25). From the logical standpoint they are indeed inseparable, although not identical, and have never been dichotomized. Psychometrically, therefore, the two measuring approaches should be regarded as adjunctive measurements.

3. *Summary*

Induction as logical process is a nondemonstrative form of argument in which the truth-value of the conclusion is not identical to the truth-value of the premises.

Nonetheless, the conclusion (statement of probability) is analytically (nonempirically) derived from the total evidence presented in the premises.

Although logically no probability adheres to the particular event in itself, epistemically we have no recourse but to indulge in prediction.

Conformance to inductive schemata in the construction of inductive tests alone makes possible the necessary convergence in terms of reasoning process between performance on the job and performance on the test.

Tests of inductive abilities are of two forms: inductive judgments of probability and generation of empirical laws which serve as premises in judgments of probability. These two forms represent different stages in a unitary process and it is extremely unlikely that one would consistently be unaccompanied by the other. Therefore, tests of inductive abilities should include both psychometric forms. The psychometric form which tests inductive abilities in the form of judgments of probability is not prominent in the psychometric literature and indeed does not exist purely as such. A test designed purely according to inductive logical schemata for judgments of probability was developed in PRDC to test these inductive abilities.

Induction and deduction as reasoning processes are closely analogous. As modes of argument they differ in terms of truth-value rather than in terms of reasoning process. (This statement applies more to the inductive derivation of a statement of probability than to the generation of the statistical law which serves as premise for this derivation.) Thus, induction and deduction should be regarded as logically inseparable (hence psychometrically adjunctive) and significant intercorrelations should be expected, and desired, between inductive and deductive subtests.

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Appendix A

AXIOMS AND THEOREMS OF THE CATEGORICAL SYLLOGISM.

The validity and nonvalidity evidences presented in the axioms and theorems of the categorical syllogism can be perceived in clear diagrammatic form through the utilization of the simple Venn diagrams method, e.g., in Quine, 1972, pp. 83-91.

The axioms of the categorical syllogism fall into two sets, those which deal with the quantity or distribution of terms[1], and those which deal with the quality of the propositions.

Axioms of Quantity

1. The middle term must be distributed at least once.
2. No term may be distributed in the conclusion which is not distributed in the premises.

Axioms of Quality

3. If both premises are negative, there is no conclusion.
4. If one premise is negative, the conclusion must be negative.
5. If neither premise is negative, the conclusion must be affirmative.

General Theorems

Theorem I:

The total number of distributed terms in the premises must exceed the number of distributed terms in the conclusion by at least one.

Proof: Axiom 2 + Axiom 1, the latter taken in conjunction with the fact that the middle term, by definition, does not appear in the conclusion.

Theorem II:

If both premises are particular, there is no conclusion.

Proof: The two particular premises may be (1) both negative, (2) both affirmative, or (3) one affirmative and one negative. In the first case Axiom 3 precludes any conclusion. In the second case, since the I proposition distributes no term, Axiom 1 precludes any conclusion. In the third case since the I proposition contains no distributed term and the O proposition contains only one distributed term, the premises contain only one distributed term. Therefore the conclusion must contain no distributed term (Theorem I). However, according to Axiom 4 the conclusion must be negative therefore containing one distributed term (O proposition). Thus the conjunction of Axiom 4 and Theorem I precludes any conclusion.

Theorem III:

If one premise is particular, the conclusion must be particular.

Proof: Since according to Theorem II the premises cannot both be particular, then they must differ in quantity and may be either (1) both negative, (2) both affirmative, or (3) one affirmative and one negative. In the first case Axiom 3 precludes any conclusion. In the second case the A proposition distributes only one term and the I proposition none. Therefore the conclusion must contain no distributed term (Theorem I) and since the A proposition contains one distributed term, the conclusion must be an I proposition. The third case includes two possible combinations: EI and AO. The EI combination yields two distributed

1. A distributed term is one which expresses the universal set and an undistributed term is one which expresses a particular subset.

terms (both in E) and the AO combination yields two distributed terms (one in A and one in O). Hence in either case the premises include a total of two distributed terms. Since according to Theorem I the conclusion must contain only one distributed term and since according to Axiom 4 the conclusion must be negative, the conclusion must be an O proposition which distributes only its predicate (the E proposition distributes both terms).

Theorem IV:

If the major premise is an I proposition and the minor an E proposition, there is no conclusion.

Proof: According to Axiom 4 the conclusion must be negative and therefore P (major term) is a distributed term in the conclusion. However the I proposition distributes no term and therefore P is undistributed in the major premise. Axiom 2 precludes this divergence in distribution.

Special Theorems

First figure

$$\begin{array}{l} M - P \\ \underline{S - M} \\ \therefore\ S - P \end{array}$$

Theorem I:

The minor premise must be affirmative.

Proof: If the minor were negative, the conclusion would be negative (Axiom 4) and P would be distributed. Hence P would be distributed in the major premise (Axiom 2) which necessitates a negative major premise. However, both premises cannot be negative (Axiom 3).

Theorem II:

The major premise must be universal.

Proof: Since the minor premise must be affirmative (Theorem I, First Figure) its predicate M is undistributed. Hence M must be distributed in the major premise (Axiom 1) thereby making it a universal.

Second Figure

$$\begin{array}{l} P - M \\ \underline{S - M} \\ \therefore\ S - P \end{array}$$

Theorem I:

The premises must differ in quality.

Proof: If both premises were affirmative M would be undistributed in each. Hence one of the premises must be negative (Axiom 1). However, both premises cannot be negative (Axiom 3).

Theorem II:

The major premise must be universal.

Proof: Since one of the premises is negative (Theorem I, Second Figure) the conclusion must be negative (Axiom 4) and P is therefore distributed in the conclusion. Hence P must be distributed in the major premise (Axiom 2) thereby making it a universal.

Third Figure

M – P
M – S
∴ S – P

Theorem I:

The minor premise must be affirmative.

Proof: If the minor premise were negative the conclusion would be negative (Axiom 4) and P would therefore be distributed in the conclusion. Hence P would be distributed in the major premise (Axiom 2) thereby necessitating a negative major premise which is made impossible by Axiom 3.

Theorem II:

The conclusion must be particular.

Proof: Since the minor premise must be affirmative (Theorem I, Third Figure), S is undistributed in the premises. Hence S cannot be distributed in the conclusion (Axiom 2) thereby necessitating its particularity.

Fourth Figure

P – M
M – S
∴ S – P

Theorem I:

If the major premise is affirmative, the minor is universal.

Proof: An affirmative major involves an undistributed M. M must therefore be distributed in the minor premise (Axiom 1) thereby necessitating its universality.

Theorem II:

If either premise is negative, the major must be universal.

Proof: If either premise is negative, the conclusion must be negative (Axiom 4) and hence its P is distributed. Hence P must be distributed in the major premise (Axiom 2) thereby necessitating its universality.

Theorem III:

If the minor premise is affirmative, the conclusion is particular.

Proof: If the minor premise is affirmative its predicate S is undistributed. Hence S cannot be distributed in the conclusion (Axiom 2) thereby necessitating its particularity.

Appendix B

EXAMPLES OF COMPOUND SYLLOGISMS IN PROPOSITIONAL FORM

(1) Compound Conditional Affirmative:

If A is B, then C is D
A is B
∴ C is D

(2) Compound Conditional Negative:

If A is B, then C is D
C is not D
∴ A is not B

(3) Pure Conditional Affirmative:

If C is D, then E is F
If A is B, then C is D
∴ If A is B, then E is F

(4) Pure Conditional Negative:

If A is B, C is not D
If E is F, C is D
∴ If A is B, E is not F

(5) Alternative:

Either A is B or C is D
A is not B
∴ C is D

(6) Disjunctive:

Not both A is B and C is D
A is B
∴ C is not D

Appendix C

PROPOSITIONAL EQUIVALENCE - DEFINITIONS

Conversion: The converse of a categorical proposition is another categorical proposition in which the original subject and predicate are reversed but in which the truth value remains the same, e. g., No S is P = No P is S. The E proposition converts simply because its terms are both distributed. Similarly, the I proposition converts simply because its terms are both undistributed. The A proposition, however, has a distributed subject and an undistributed predicate. Since the original P must remain undistributed as the subject of the converse, the proposition must be reduced in quantity to particularity: All S are P = Some P are S. Lastly, the O proposition has no converse because the distribution of S would be universalized in the converse.

Obversion: The obverse of a categorical proposition is another categorical proposition in which the original predicate is negated and the quality of the original proposition is changed, e. g., All S are P = No S is non-P. The truth value remains the same. The four types of categorical propositions can be obverted.

Obverted converse: obversion of the converse of a categorical proposition, e.g., No S is P = No P is S = All P are non-S. Since the O proposition has no converse, it has no obverted converse.

Contraposition: The full contrapositive of a categorical proposition is another categorical proposition in which the subject is the original predicate in negative form and the predicate is the original subject in negative form. It is obtained through a series of obversion-conversion-obversion, e.g., No S is P = All S is non-P = Some non-P are S = Some non-P are not non-S. The I proposition has no contrapositive because its obverse is an O proposition which has no converse.

Inversion: The full inverse of a categorical proposition is another categorical proposition in which the subject is the original subject in negative form, the predicate is the original predicate, and the quality of the proposition is changed. It is obtained through a series of obversion-conversion-obversion-conversion-obversion, e.g., All S are P = No S is non-P = No non-P is S = All non-P are non-S = Some non-S are non-P = Some non-S are not P. The I and O propositions do not yield full inversion.

Compound propositions can also be analyzed in terms of propositional equivalence. For example, If p then q has no converse since the fact that p implies q does not tell us whether q implies p. However, it clearly converts legitimately to its contrapositive: If non-q then non-p.

Nonetheless we may have the case, e.g., in a mathematical definition, in which both If p then q and If q then p are true. This is called a biconditional statement which expresses the concept *p if and only if q*. In expressing this concept it expresses biconditionality, i.e., the statement *p if and only if q* means that If q then p and that If ∼q then ∼p; but since If ∼q then ∼p is the contrapositive equivalent of If p then q, then *p if and only if q* is a biconditional statement which expresses both If q then p and If p then q. Symbolically the biconditional is expressed as p<->q.

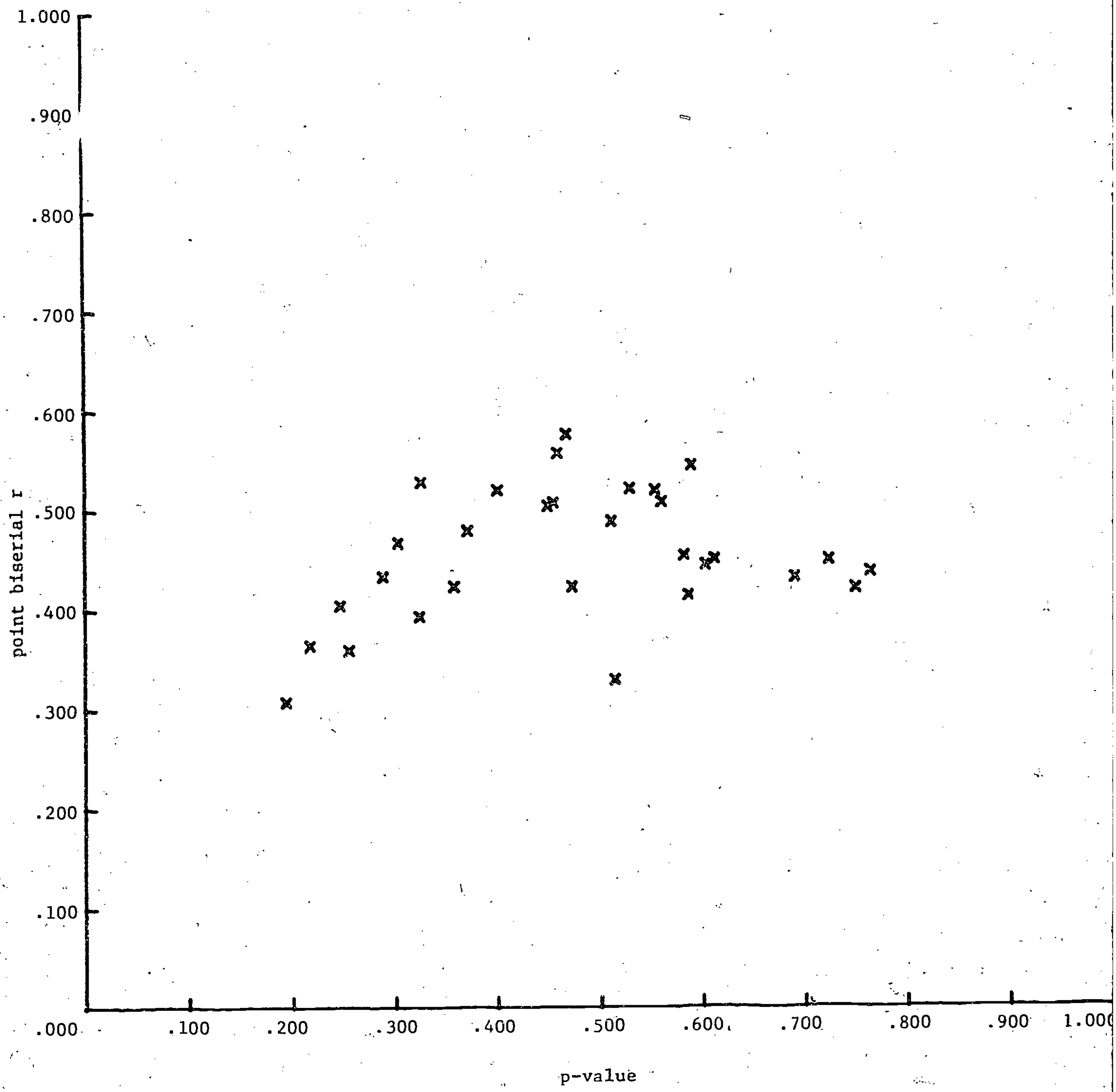
Appendix D

Frequency distribution of p-value (proportion correct)
for Series 21013: Modified Deduction

p-value interval	f	% of total
.900-.999	0	0.0
.800-.899	0	0.0
.700-.799	3	10.0
.600-.699	3	10.0
.500-.599	8	26.7
.400-.499	6	20.0
.300-.399	5	16.7
.200-.299	4	13.3
.100-.199	1	3.3
.000-.099	0	0.0

Frequency distribution of point biserial r
for Series 21013: Modified Deduction

point biserial r interval	f	% of total
.550-.599	2	6.7
.500-.549	8	26.7
.450-.499	6	20.0
.400-.449	9	30.0
.350-.399	3	10.0
.300-.349	2	6.7
.250-.299	0	0.0
.200-.249	0	0.0
.150-.199	0	0.0
.100-.149	0	0.0



Scatterplot of p-values and point biserial r's
for Series 21013: Modified Deduction

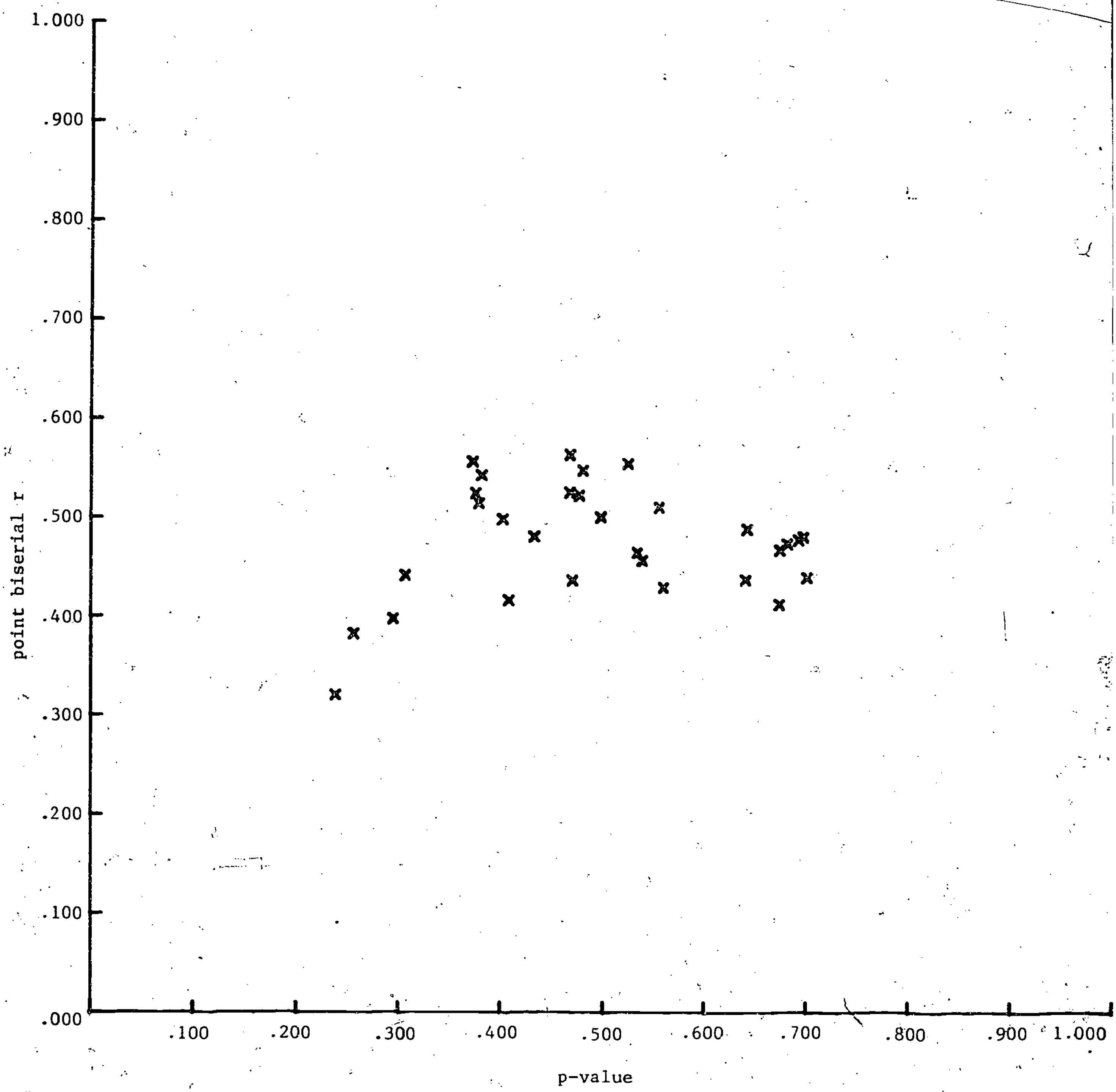
Appendix E

Frequency distribution of p-value (proportion correct)
Series 21014: Modified Deduction

p-value interval	f	% of total
.900-.999	0	0.0
.800-.899	0	0.0
.700-.799	1	3.3
.600-.699	7	23.3
.500-.599	5	16.7
.400-.499	9	30.0
.300-.399	5	16.7
.200-.299	3	10.0
.100-.199	0	0.0
.000-.099	0	0.0

Frequency distribution of point biserial r
for Series 21014: Modified Deduction

point biserial r interval	f	% of total
.550-.599	3	10.0
.500-.549	8	26.7
.450-.499	9	30.0
.400-.449	7	23.3
.350-.399	2	6.7
.300-.349	1	3.3
.250-.299	0	0.0
.200-.249	0	0.0
.150-.199	0	0.0
.100-.149	0	0.0



Scatterplot of p-values and point biserial r's
for Series 21014: Modified Deduction

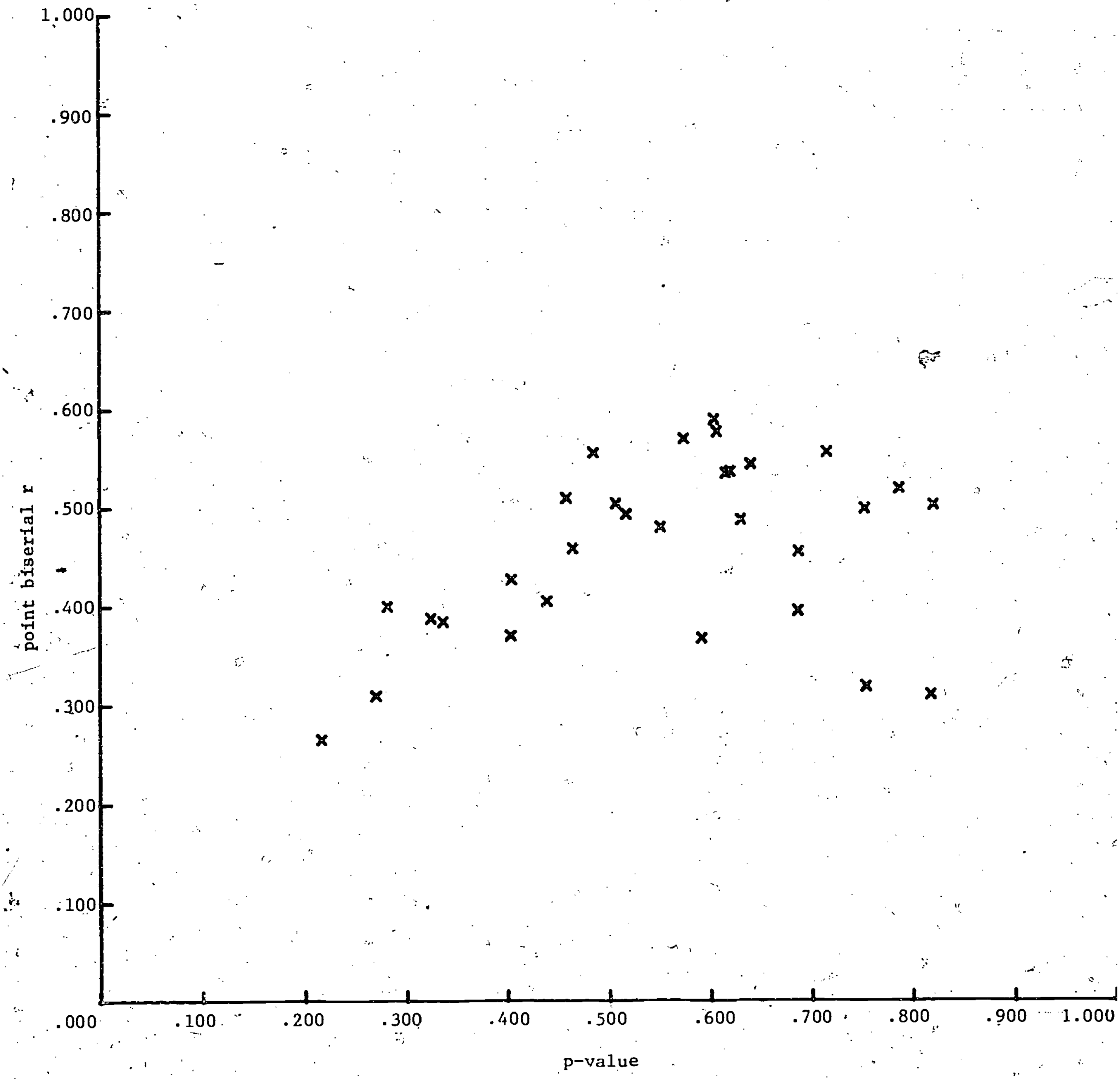
Appendix F

Frequency distribution of p-value (proportion correct) for Series 19012: Modified Deduction

p-value interval	f	% of total
.900-.999	0	0.0
.800-.899	2	6.7
.700-.799	4	13.3
.600-.699	8	26.7
.500-.599	5	16.7
.400-.499	6	20.0
.300-.399	2	6.7
.200-.299	3	10.0
.100-.199	0	0.0
.000-.099	0	0.0

Frequency distribution of point biserial r for Series 19012: Modified Deduction

point biserial r interval	f	% of total
.550-.599	5	16.7
.500-.549	7	23.3
.450-.499	6	20.0
.400-.449	3	10.0
.350-.399	5	16.7
.300-.349	3	10.0
.250-.299	1	3.3
.200-.249	0	0.0
.150-.199	0	0.0
.100-.149	0	0.0



Scatterplot of p-values and point biserial r's
for Series 19012: Modified Deduction

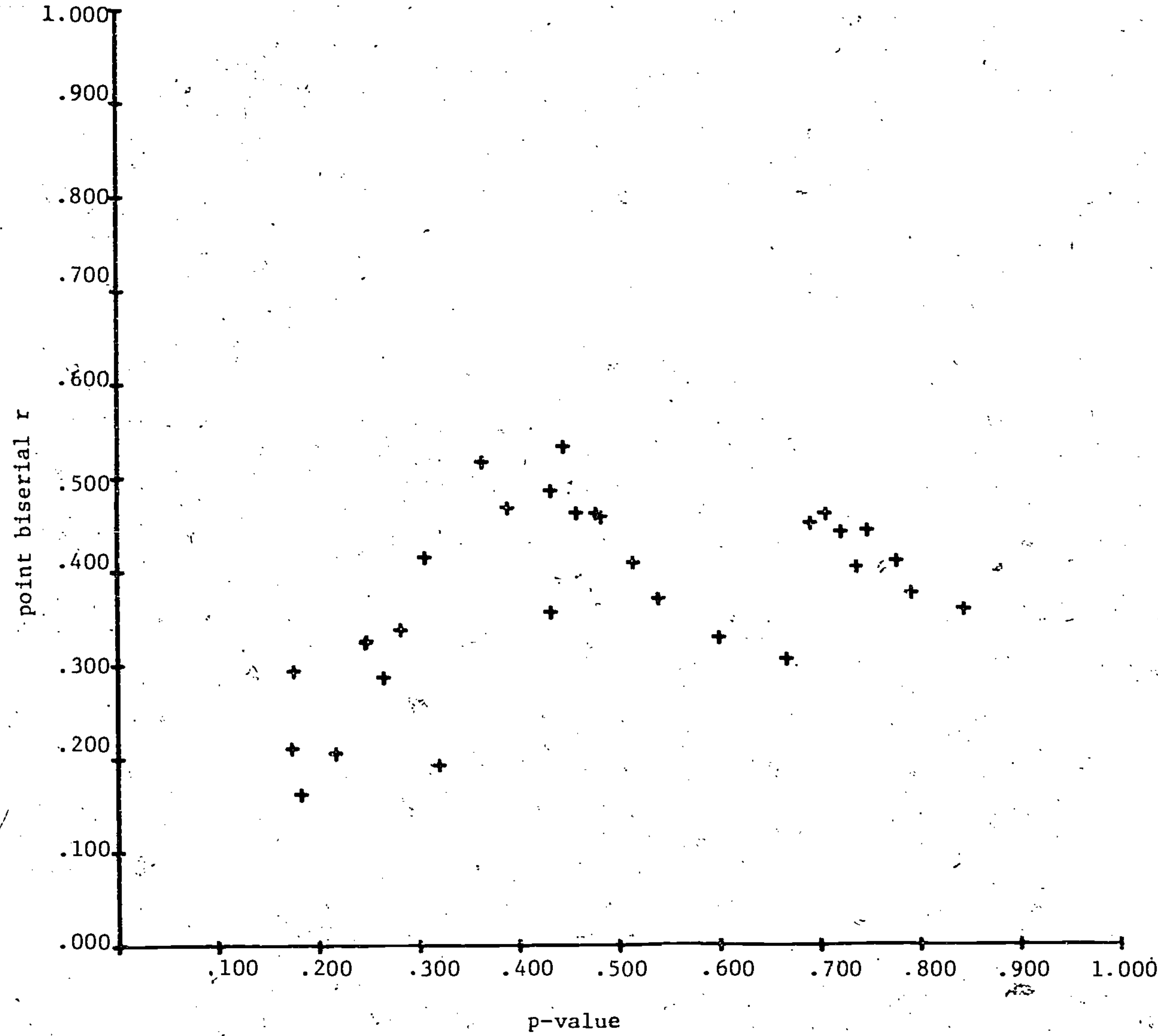
Appendix G

Frequency distribution of p-value (proportion correct) for 30-question series of inductive judgments of probability (Series 21015)

p-value interval	f	% of total
.900-.999	0	0.0
.800-.899	1	3.3
.700-.799	6	20.0
.600-.699	3	10.0
.500-.599	2	6.7
.400-.499	6	20.0
.300-.399	4	13.3
.200-.299	5	16.7
.100-.199	3	10.0
.000-.099	0	0.0

Frequency distribution of point biserial r for 30-question series of inductive judgments of probability (Series 21015)

point biserial r interval	f	% of total
.500-.549	2	6.7
.450-.499	7	23.3
.400-.449	6	20.0
.350-.399	4	13.3
.300-.349	5	16.7
.250-.299	2	6.7
.200-.249	2	6.7
.150-.199	2	6.7
.100-.149	0	0.0
.050-.099	0	0.0
.000-.049	0	0.0



Scatterplot of p-values and point biserial r's
for 30-question series of inductive judgments of probability (Series 21015)

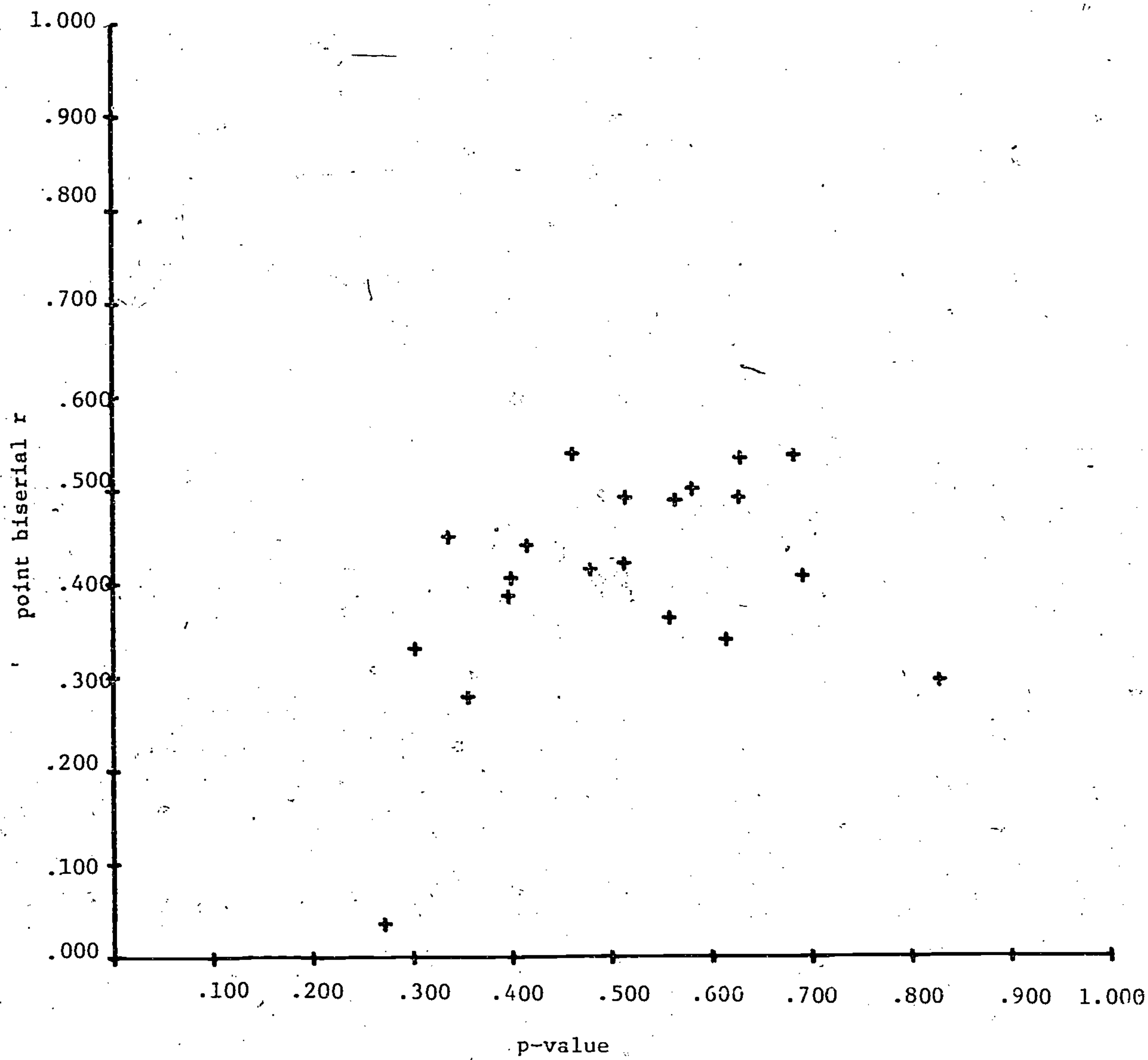
Appendix H

Frequency distribution of p-value (proportion correct) for 20-question series of inductive judgments of probability (Series 26006)

p-value interval	f	% of total
.900-.999	0	0.0
.800-.899	1	5.0
.700-.799	0	0.0
.600-.699	5	25.0
.500-.599	5	25.0
.400-.499	3	15.0
.300-.399	5	25.0
.200-.299	1	5.0
.100-.199	0	0.0
.000-.099	0	0.0

Frequency distribution of point biserial r for 20-question series of inductive judgments of probability (Series 26006)

point biserial r interval	f	% of total
.500-.549	4	20.0
.450-.499	4	20.0
.400-.449	5	25.0
.350-.399	2	10.0
.300-.349	2	10.0
.250-.299	2	10.0
.200-.249	0	0.0
.150-.199	0	0.0
.100-.149	0	0.0
.050-.099	0	0.0
.000-.049	1	5.0



Scatterplot of p-values and point biserial r's
for 20-question series of inductive judgments of probability (Series 26006)

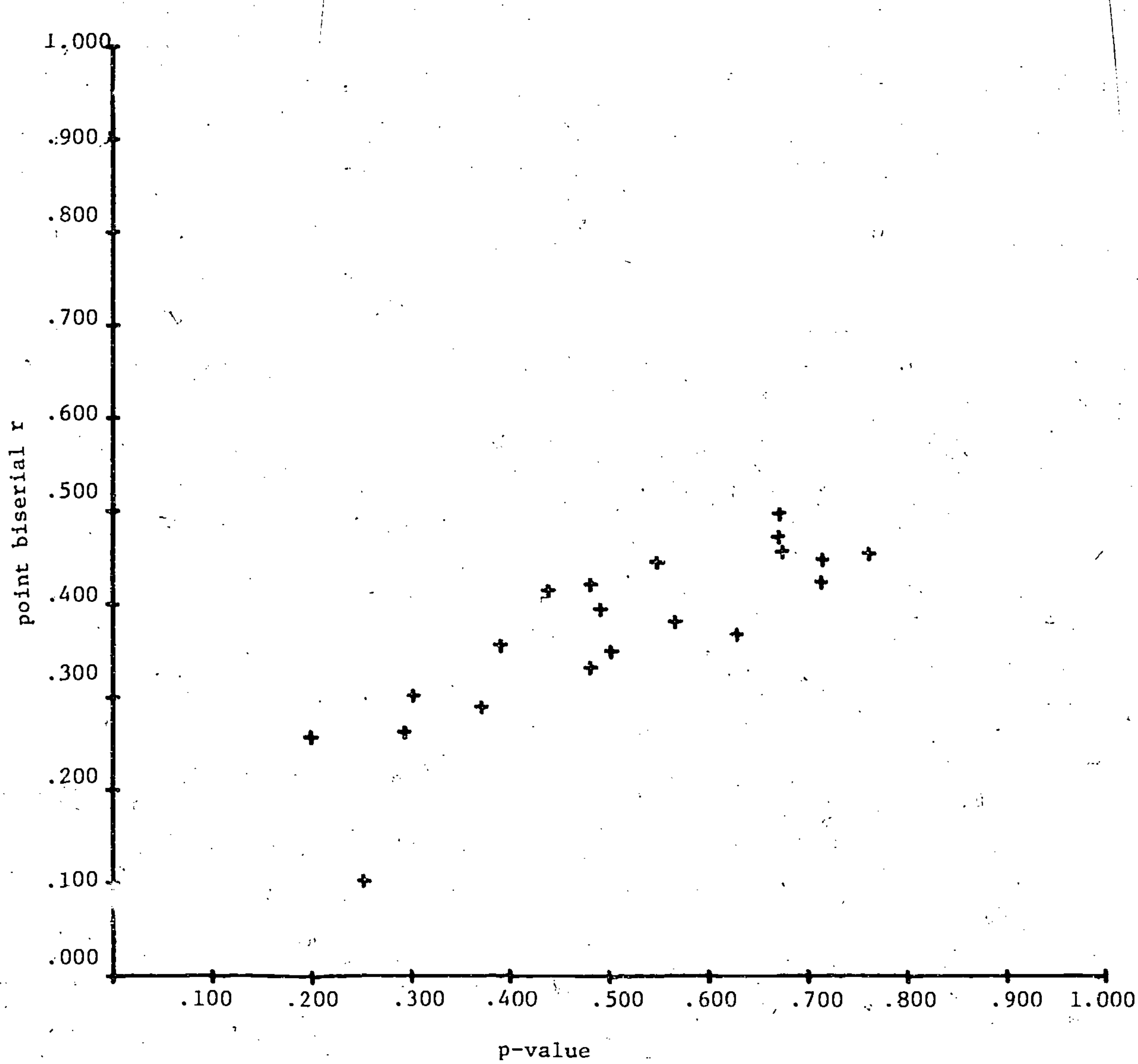
Appendix I

Frequency distribution of p-value (proportion correct) for 20-question series of inductive judgments of probability (Series 26007)

p-value interval	f	% of total
.900-.999	0	0.0
.800-.899	0	0.0
.700-.799	3	15.0
.600-.699	4	20.0
.500-.599	3	15.0
.400-.499	4	20.0
.300-.399	3	15.0
.200-.299	2	10.0
.100-.199	1	05.0
.000-.099	0	0.0

Frequency distribution of point biserial r for 20-question series of inductive judgments of probability (Series 26007)

point biserial r interval	f	% of total
.500-.549	0	0.0
.450-.499	4	20.0
.400-.449	5	25.0
.350-.399	5	25.0
.300-.349	2	10.0
.250-.299	3	15.0
.200-.249	0	0.0
.150-.199	0	0.0
.100-.149	1	5.0
.050-.099	0	0.0
.000-.049	0	0.0



Scatterplot of p-values and point biserial r's
for 20-question series of inductive judgments of probability (Series 26007)